Surveying FE 208
Lecture 2

Errors in Measurements

Learning Objectives for this Lecture

1. Define *Direct* and *Indirect* measurements
2. Define *Errors* and *Mistakes* in measurements
3. Be able to write the equation for errors
4. Know the four rules for all measurements
5. Know the four error sources
6. Know the two error types
7. Be able to calculate the adjustment for systematic errors
8. Define *Accuracy*, *Precision*, and *Bias*

Two types of measurements

Direct measurements – Measurements made first-hand using some type of measuring device or instrument in a direct manner.

Examples include:

Ground distance with a tape
Tree diameter with a Spencer tape or caliper
Level elevation with a level gun and level rod

Indirect measurements – Measurements made using an observable proportion or ratio

Examples include:

Tree height using a shadow and a known pole and shadow
Can be set up as a ratio, for example

$$\frac{\text{Pole Height}}{\text{Pole Shadow Length}} = \frac{\text{Tree Height}}{\text{Tree Shadow Length}}$$

$$\frac{25'}{35'} = \frac{\text{Tree Height}}{142'} = 101.4'$$

Errors in Measurements

Measurements are Inexact

Even with the most sophisticated equipment, a measurement is only an estimate of the true size of a quantity. Exactness simply does not exist in the physical world. "Truth" is always elusive when it comes to measurement. This is because the instruments, as well as the people using them are imperfect, because the environment in which the instruments and people operate influences the process, and because the behavior of people, instruments, and the environment cannot be fully predicted.

How to approximate the degree of inexactness of a reading (or "observation") is a fundamental concern of any professional surveyor. It should also be of concern to others who use measured data, since any misunderstanding could lead a person or group of people to wrong conclusions. If measured data is to be interpreted and used correctly, the concept of inexactness must never be denied.

Errors

Errors are defined as the difference between a measured value and the true value

$$e_i = x_i - \mu_i$$

where:

e_i = Individual error in a single measurement
x_i = Individual measurement value
μ_i = True value of the measurement

We know four things regarding measurements and measurement errors:

1. No measurement is exact.
2. All measurements have error.
3. μ is never known (because of number 2).
4. Exact error is never known (Because of number 3).

Therefore, we can estimate errors and estimate confidence in values for the estimate of μ

Note that x_i is an estimate of μ

Mistakes

Mistakes are ___not___ errors. They are also called blunders

Caused by things like:

- Poor technique
- Carelessness
- Fatigue
- Poor communication
- Misreading an instrument

Large mistakes can often be spotted in the field notes, small mistakes are hard to detect

Error Source

There are three types of errors related to measurements:

1. Natural Errors – Errors caused by some variability in natural conditions such as temperature – Stretch in a steel tape, Pressure – measurements in a total station, wind-holding a level rod straight, magnetic declination – variation in a compass

2. Instrument Errors – Errors caused by imperfections in the manufacture of an instrument.

3. Personal Error- The error caused by the inability to line up crosshairs exactly on a measurement point or sight exactly on a line

Error Types

There are two types of errors related to measurements:

1. Systematic Errors – Also called bias. Errors that are constant and also are cumulative. Always in a single direction and can be compensated for if detected. For example, a compass that is off by 5 degrees or a tape that is missing 1 foot in its length

A tape that measures 99.94' between the 0 mark and the 100' mark when compared against a standard, would consistently be off by 0.06' every time a 100' measure is taken. For example, if a 300' measurement was made with the tape, the total distance would be off, systematically, by 0.06' for each of the 100' sections, for a total systematic error of 0.18'. The error would always be off in the same direction, 0.06' short. If we knew ahead of time that the tape was short, we could compensate for it by adding the 0.06' each time.

Systematic errors can be adjusted by the equation:

$$T = R + C$$

Where:
T = the true value
R = the instrument reading
C = a calculated correction value

Example:

A 150' steel tape is used to measure a known 100-foot distance between two points on the ground. Four repeated measurements yield the following:

Measure 1 = 100.2'
Measure 2 = 200.4'
Measure 3 = 300.6'
Measure 4 = 400.8'

The average measurement of the four is 400.8/4 = 100.2'. This represents a systematic error of 0.2' per 100'. Afterwards it is found that the tape was 0.15 feet longer than its expressed length. This error is calculated to be 0.1' per 100 feet (0.15'/150').

A new measurement is taken between two points and found to be 723.8'. Accounting for the systematic errors above, the true value of the measurement is:

T = 723.8' + (-0.1 * 7.238) + (-0.2 * 7.238)
T = 723.8' + (-2.2')
T = 721.6'

2. Random Errors – These errors follow a natural distribution and tend to be offsetting. Random errors have a magnitude and sign that we cannot compensate for.

A tape is held over a small nail set in a wooden hub point at each end. The true value of the point is exactly center of the nail but because we cannot know the exact center, the tape will sometimes be to the right and sometimes to the left (sign) and the exact measure to the center will be slightly long or short (magnitude)

Random error is the error we calculate statistics for.

Random errors are propagated through a survey by theory through the distribution of the errors in *series*. Buckner (1983) defines error propagation as the mathematical process used to estimate the expected random error in a computed quantity, caused by one or more random errors in one or more identified variables. In other words, for each variable identified in a measurement process, a random error will occur for each that contributes to the overall error. The total error is calculated as:

$$e_Y = \sqrt{e_{X1}^2 + e_{X2}^2 + e_{X3}^2 + ...e_{Xn}^2}$$

Where:
e_Y = Total error also called the *standard error of the total*
e_{Xn} = Error accounted for by variable X_n also called the *standard error of the variable*

Accuracy vs. Precision

Accuracy (the lack of systematic mistakes) is how close a measurement or observation is to the true value. In statistics, it usually refers to how close the sample *mean* (a statistic) is to the true *mean* (a parameter).

Precision has two connotations. One is how many decimals of precision exist. Thus, a measurement of 8.4079 ft is said to be more precise than 8.4 ft.

The other connotation (one used in statistics) is how reproducible are the measurements. For example, suppose we measured something 100 times and our measurements ranged from 8.3412 to 8.9726. Suppose we take a different set of 100 measurements that ranged from 8.4070 to 8.4079. Our second set of 100 measurements is more precise. In statistics, precision is usually expressed in terms of the standard deviation or standard error. It is important to note that a set of measurements can be precise, but not accurate and vice versa.

a.   b.   c.
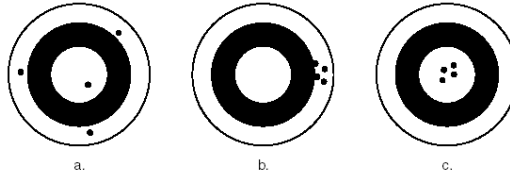
These figures show the differences in the terms "accuracy," "precision," and "bias." The points on target (a) show results that are neither accurate nor precise. In (b), the points on the target are precise in that they are closely grouped, but are not accurate. These points are biased. In (c), the points are both precise and accurate.

Reading for this Section

Moffitt and Bossler, pages 11-13
Kiser, pages 2-8
Buckner, Part 1-Part 4