Online Surveying FE 208
Lecture 4

Fundamentals of Survey Measurements

Learning Objectives for this Lecture

1. **Have a basic understanding of survey units**
2. **Know and be able to convert survey units to chains**
3. **Be able to work with correct significant figures**
4. **Know the four dimensions of survey measurements**
5. **Know the basic trigonometric relationships**
6. **Know the angle signs for the quadrants**
7. **Know the conversions for degree angles to radian angles**
8. **Know the conversion for slope percent to slope degree angle**

Survey Units of Measure

The United States is one of only three countries still using the English system of measurements.
All other countries are metric or SI (International System of Measurements)

<u>English System – Horizontal and Vertical Distance</u>

Foot – The basic unit of measure

1893 – Adoption of the ***survey foot***

39.37 inches = 1 meter
1 foot = 0.3048006 meter

1959 – Adoption of the ***international foot.*** Slightly shorter.

1 inch = 2.54 centimeters
1 foot = 0.3048 meter

Difference is about 1 foot in 100 miles

U.S. public lands were surveyed prior to 1959. Thus, the old standard still applies.

Other common survey units:

Rod - 16 ½ feet (Also called a pole or perch)

***Chain –** 66 feet . 100 links in a chain. 4 rods in a chain. Also called a Gunter's chain.
10 square chains = 1 acre.
Edmund Gunther 1581-1626 English Mathematician.
Invented cosine and cotangent terms and discovered magnetic variation.

Mile – 80 chains.
Vara – about 33 inches. Old Spanish unit. Primarily used in SW U.S. and Mexico
Arpent – 191.8 feet. Old French unit. Primarily in Louisiana and Canada

**This is one of the most important units used by foresters*

English System – Horizontal and Vertical Angles

Degree – basic unit of measure for angles. 360 degrees in a circle.

Minute – 60 minutes in a degree.

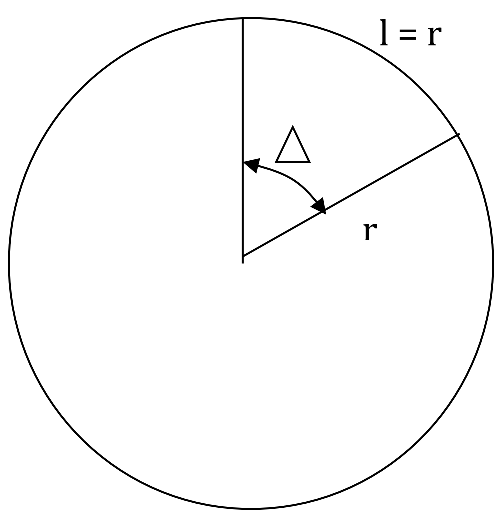
Second – 60 seconds in a minute.

Others:

Grads – 400 grads to a circle (also called gons)

Radian – The angle subtended by an arc of a circle having a length = radius.

$2\,\pi\,r$ = circumference
$2\,\pi$ rad = 360°
1 rad = 57°17'44.8"



Significant Figures

Definition – The number of digits known plus one estimated or rounded off digit.

Requirement is that you know the precision of the measurement instrument. The significant figures used then are the $\pm$ last digit of the instrument precision.

<u>Example</u>

A steel tape with graduations to the 0.01' is used to make four repeat measurements of a distance. The recorded values are as follows:

100.04' 100.07' 100.06' 100.10'

The average of these record measurements is the sum divided by four.

$$= 400.27 / 4$$

On your calculator, this can be expressed as 100.0675. However, this falsely implies that your measurement was precise to 0.0001'. The correct answer is expressed as 100.07.

The digit 7 is the rounded off or estimated value. This number is said to have 5 significant figures.

Introduction to Surveying Measurements

Surveying processes are generally divided into two categories, geodetic and plane. The primary difference between geodetic and plane surveying is the reference that they use.

Geodetic surveying

Geodetic surveying measures all elevations from a level surface, and because all instruments use a straight reference plane, this requires all instrument readings of elevation to be adjust for the curvature of the earth. Geodetic surveys are used when a high degree of accuracy is needed and when the survey will cover long distances or a large area.

Plane surveying

Plane surveying assumes the earth is flat and all elevations are measured from a flat surface (plane). In plane surveying every elevation measurement has a small amount of error, however plane surveying is easier to complete and provides sufficient accuracy for smaller areas.

The basic unit in surveying is the ***point***. All surveying measurements are made between points that have been located or established. The survey methods used to make measurements and determine direction and position between points are based on the following definitions and techniques:

Measurement of Dimensions

Four dimensions are measured: Horizontal lengths, vertical lengths, horizontal angles, and vertical angles.

Horizontal Length

Horizontal length is the ***straight line distance measured in a horizontal plane***. In most cases horizontal distance is calculated from a distance measured on a slope. Measurements are made by direct and indirect methods. Direct measurements are made by wheels, human pacing, and tapes of cloth, metallic cloth, or steel. Indirect measurements are typically made by use of electronic distance-measuring equipment (EDM). EDM's vary from inexpensive laser devices that measure distance only to those that are part of more complex instruments such as total stations.

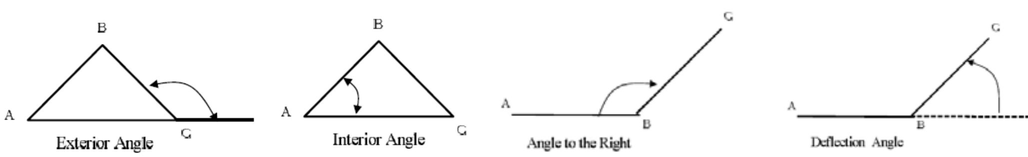
Vertical Length

A vertical length is a measurement of a ***difference in height or elevation***. Direct measurements can be made by an altimeter, which indicates barometric pressure or by a plumb line and tape for short vertical distances. In most cases remoteness of points and accuracy require indirect measurements with instruments such as the level and graduated rod or total station or transit. Some EDM's have the capability to measure vertical distance as well as horizontal distance.

Horizontal Angle

A horizontal angle is the difference in direction between: ***(1) two intersecting lines in a horizontal plane; (2) two intersecting vertical planes; or (3) two intersecting lines of sight to points in different vertical planes***. It is measured in the horizontal plane in *degrees of arc*. Horizontal angles usually are measured clockwise (called an angle-right) but may be measured counterclockwise (angle-left).

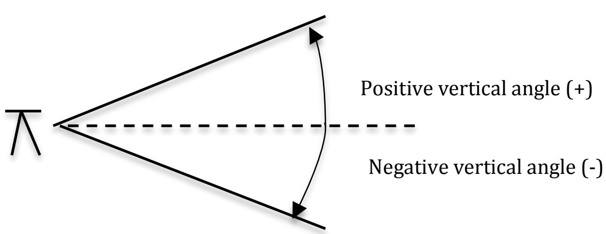
Angles are usually measured directly by total station, theodolite, or transit. An interior angle is on the inside of an enclosing figure, and an exterior angle is on the outside of an enclosing figure. A deflection angle is that angle which any line extended makes with the succeeding or forward line. The direction of the deflection is identified as “right” or “left.” An angle-to-the-right is the clockwise angle at the vertex between the back line and forward line



Positive vertical angle

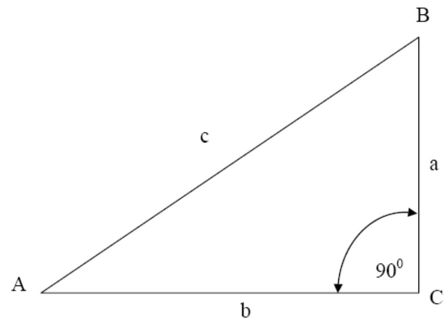
Vertical Angle

A vertical angle is the ***difference in direction between a horizontal plane and an intersecting line, plane, or a line of sight to a point***. It is measured in the vertical plane in *degrees of arc*. Measurements are referenced “up” or “down” from the horizontal as “plus angles” or “minus angles”



<u>Basic Surveying Mathematics</u>

There are basic trigonometric relationships that are used in plane surveying to define angles and their relationships to each other. In most instances, field surveying problems can be constructed in the form of right triangles. The trigonometric functions of right triangles are defined by the ratios of the lengths of the triangle sides to the angles of the triangle. For the field surveyor these can be reduced to a simple set of functions shown in the example below.



The primary trigonometric functions used by surveyors are the *sine* (sin), *cosine* (cos), and *tangent* (tan) along with the *Pythagorean Theorem*. The general relationships are:

$$\text{sin angle} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

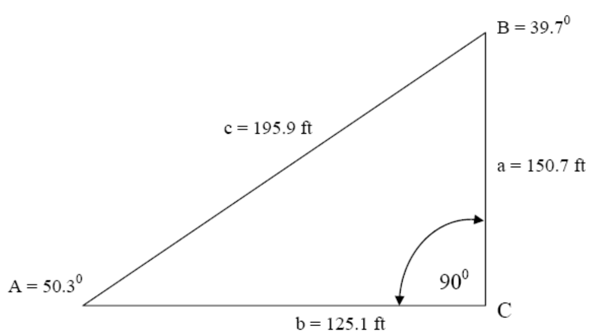
$$\text{cos angle} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{tan angle} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{Pythagorean Theorem} = a^2 + b^2 = c^2$$

$$A + B + C = 180^0$$

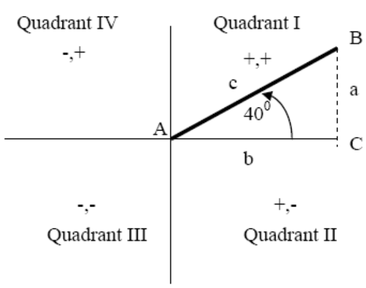
We are only concerned with 6 relationships since angle C in a right triangle is always known. Table 2-1 shows these relationships for the right triangle in figure 2-3 with example calculations.



Function	Relationship	Example calculation
c	$= \sqrt{b^2 + c^2}$	$= \sqrt{125.1^2 + 150.7^2} = 195.9$
a	$= \sqrt{c^2 - b^2}$	$= \sqrt{195.9^2 - 125.1^2} = 150.7$
b	$= \sqrt{c^2 - a^2}$	$= \sqrt{195.9^2 - 150.7^2} = 125.1$
sin A	$= \frac{a}{c}$	$= \frac{150.7}{195.9} = 0.7693;\ A = \sin^{-1} 0.7693 = 50.3^0$
sin B	$= \frac{b}{c}$	$= \frac{125.1}{195.9} = 0.6386\ \ B = \sin^{-1} 0.6386 = 39.7^0$
cos A	$= \frac{b}{c}$	$= \frac{125.1}{195.9} = 0.6386\ \ A = \cos^{-1} 0.6386 = 50.3^0$
cos B	$= \frac{a}{c}$	$= \frac{150.7}{195.9} = 0.7693;\ B = \cos^{-1} 0.7693 = 39.7^0$
tan A	$= \frac{a}{b}$	$= \frac{150.7}{125.1} = 1.2046;\ A = \tan^{-1} 1.2046 = 50.3^0$
tan B	$= \frac{b}{a}$	$= \frac{125.1}{150.7} = 0.8301;\ B = \tan^{-1} 0.8301 = 39.7^0$

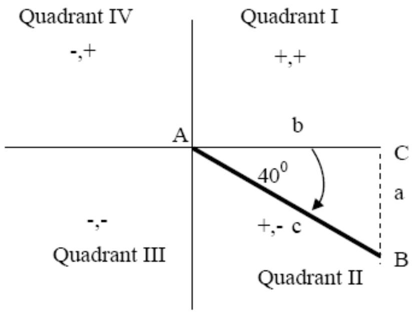
Another function of angles is that they can be considered as positive or negative. The figures below show the relationship of any angle to the quadrant of a circle and the trigonometric relationship of that angle.

<u>NOTE: It is important to note that the trigonometric functions are based on the angle direction from the horizontal axis. In all surveying, the angles will originate from the vertical or North-South axis</u>



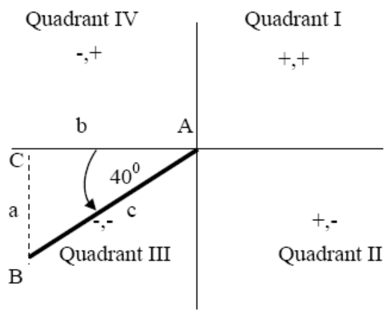
Quadrant I

Line	Direction	sin a\c	cos b/c	tan b/a
a	+	+	+	+
b	+			
c	+			



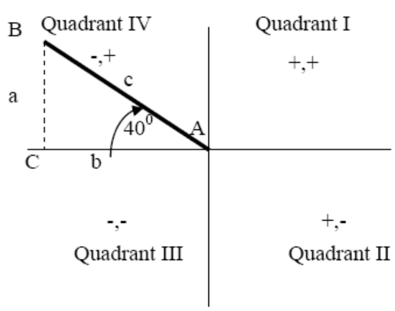
Quadrant II

Line	Direction	sin a\c	cos b/c	tan b/a
a	-	+	-	-
b	+			
c	-			



Quadrant III

Line	Direction	sin a\c	cos b/c	tan b/a
a	-	-	-	+
b	-			
c	+			



Quadrant IV

Line	Direction	sin a\c	cos b/c	tan b/a
a	+	-	+	-
b	-			
c	-			

Conversion of Degrees to Decimal Equivalents

It is important to note that anytime trigonometric functions are used, all bearings or azimuths must be converted from degrees-minutes-seconds to their decimal equivalent. Some calculators can do this automatically. This can also be done easily as the following example shows.

To convert 35^0 17’ 23” to the decimal equivalent:

a. Degrees do not change.

35^0

b. Divide the seconds by 60 to get the fraction of a minute and add the whole minutes to this.

$$23/60 + 17 = 17.38$$

c. Divide this value by 60 to get the fraction of a degree and add to the whole degrees.

$$17.38/60 + 35 = 35.2897^0$$

To convert back to degrees-minutes-seconds:

d. Degrees do not change.

$$35^0$$

e. Multiply the decimal number remaining by 60. The whole number is the minutes.

$$0.2897 * 60 = 17.382 = 35^0\ 17' + 0.382$$

f. Again, multiply the decimal number remaining by 60. This is the seconds.

$$0.382 * 60 = 23'' = 35^0\ 17'\ 23''$$

Conversion of Degrees to Radian Equivalents

The radian is the standard unit of angular measure, used in many areas of mathematics. An angle's measurement in radians is numerically equal to the length of a corresponding arc of a unit circle, so one radian is just under 57.3 degrees

Often it will be necessary to convert degrees to radians. The formula for the conversion is:

Angle in radians = (angle in degrees) * (Pi/180)

To go back to degrees:

Angle in degrees = (angle in radians) / (Pi/180)

Example:
For a 120^0 angle;

Angle in radians = 120^0 * (Pi/180) = 2.09 radians

Working with slope angles in percent

A last important function in surveying mathematics is the conversion of slope angle (vertical angle) from percent to degrees. Slope angle in surveying is usually measured with an instrument called a clinometer (or inclinometer) and is measured as a percentage.

Remember from basic math that a percentage is also expressed as rise or run or vertical distance over horizontal distance.

From the table above, note that the Angle A is the inverse tangent ($\tan^{-1}$) of the rise over run (or percentage slope)

Example: Convert a slope angle of 36% to a vertical angle

Vertical angle = $\tan^{-1}(0.36) = 19.80$ degrees

<u>Reading for this Section</u>
Kiser, pages 29-36